

Supersymmetry of classical solutions in Chern-Simons higher spin supergravity

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ABSTRACT: We construct and study classical solutions in Chern-Simons supergravity based on the superalgebra $sl(N|N-1)$. The algebra for the $N=3$ case is written down explicitly using the fact that it arises as the global part of the superconformal \mathcal{W}_3 superalgebra. For this case we construct new classical solutions and study their supersymmetry. Using the algebra we write down the Killing spinor equations and explicitly construct the Killing spinor for conical defects and black holes in this theory. We show that for the general $sl(N|N-1)$ theory the condition for the periodicity of the Killing spinor can be written in terms of the products of the odd roots of the superalgebra and the eigenvalues of the holonomy matrix of the background. We verify this explicitly for the higher spin conical defects constructed in the $sl(3|2)$ theory.

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1. Introduction

Consistent theories of higher spin fields interacting with gravity constructed by Vasiliev [1] (see [2] for a review) have been the focus of many recent works. These theories are interesting from the perspective of the AdS/CFT since they are examples of gravitational backgrounds in which one does not need to deal with the entire spectrum of massive string excitations but only with infinite set of higher spin massive fields. Higher spin theories on AdS_4 have been proposed as dual descriptions of vector like field theories [3, 4, 5], sub-sectors of free Yang-Mills theories [6, 7, 8, 9], and very recently argued to be duals of certain ABJ models [10].

Higher spin theories in three spacetime dimensions are particularly tractable since in this situation the Vasiliev like theories can be formulated in terms of a Chern-Simons theory [11]. Furthermore in three dimensions it is not necessary to consider an infinite number of higher spin fields to obtain consistent interactions. It is possible to work with a finite set of higher spin fields. Vasiliev like theories in 3

dimensions coupled to a massive complex scalar have been proposed to be holographic duals to \mathcal{W}_N minimal models based on the coset [12]

$$\frac{SU(N)_k \otimes SU(N)_1}{SU(N)_{k+1}}. \quad (1.1)$$

This duality is a new example for the AdS_3/CFT_2 correspondence and various checks of the proposal include matching of the symmetries, comparison of their one loop partition function and three point correlators (see [13] for a comprehensive list of references). A supersymmetric extension of the higher spin/minimal model duality has been proposed in [14]. This duality has also been checked by comparison of the symmetries and the partition function [15, 16]. Chern-Simons theories based on super-extended higher spin superalgebras have been considered in [17] and their asymptotic algebras have been shown to agree with the corresponding superconformal \mathcal{W}_∞ algebra.

Studying classical solutions in Chern-Simons theories based on a higher spin group provides more more insights to holography in three dimensions. The higher spin black holes found in [18] and the conical defect solutions [19] have proved to be useful to study aspects of the holographic renormalization group, nature of singularities in higher spin gravity [20, 21] and also served as candidates of light states of the dual CFT.

Motivated by these developments we study and construct new classical solutions in Chern-Simons theories based on the $sl(N|N-1)$ superalgebra. Our working example will be the algebra $sl(3|2)$ which is the global part of the \mathcal{W}_3 superalgebra in the large central charge limit. All these theories have two $U(1)$ gauge fields corresponding to the R symmetry of the dual conformal field theory. Studying and classifying supersymmetric solutions in these models will help to understand the nature of chiral primaries of the dual CFT which is important in verifying aspects of the supersymmetric higher spin/minimal model correspondence. The supersymmetric conditions in the Chern-Simons theory based on supergroups which contained spins ≤ 2 were earlier analyzed in [22, 23, 24, 25, 26]. Once the background flat connection is given, the Killing spinor equations are particularly easy to write in in a Chern-Simons theory based on a any supergroup. The Killing spinor equation is just a covariant derivative with the flat connection as the background. Thus knowing the supergroup structure is sufficient to write down the Killing spinor equation. By studying various solutions we arrive at the observation that the solution admits a periodic Killing spinor if the combined $U(1)$ part of the holonomy together with the holonomy of the rest of the connection along around the angular direction is trivial. This observation enables us to state the condition on the periodicity of the Killing spinor in terms of the odd roots of the $sl(N|N-1)$ superalgebra and the eigen values of the holonomy matrix.

The organization of this paper is as follows: In the next section we review some generalities of higher spin AdS_3 supergravity and write down the Killing spinor equation for any Chern-Simons theory based on a given super group. We then provide the details of the commutation relations for the $sl(3|2)$ superalgebra. We derive them by considering the global part and the large central charge limit of the super \mathcal{W}_3 conformal algebra written down in [27]. In section 3 we study the supersymmetry of various classical solutions in the Chern-Simons based on the $sl(3|2)$ superalgebra. These include the BTZ black hole, the black hole with higher spin field and also a new black hole solution with fields corresponding to the $sl(2)$ required for the supersymmetric completion of the bosonic $sl(3)$ turned on. We then study the supersymmetry of conical defects in these theories. Again these defects also include those with fields in the $sl(2)$ turned on. In section 4 we show that the periodicity requirement of the Killing spinor in the angular direction can be cast in terms of the holonomies of the background flat connection. We show that the supersymmetric conditions of any background can be written in terms of products of the odd roots of the superalgebra with eigen values of the holonomy matrix of the background. Section 5 contains the conclusions and a discussion of the results.

Note added: After completion of this work, we received [28] which overlaps with some portions of this paper.

2. Chern-Simons higher spin supergravity

It is well known that pure gravity in AdS_3 can be written in terms of difference of two Chern-Simons actions based on the algebra $sl(2, R)$ [29]. Similarly super symmetric extensions of pure gravity containing spins ≤ 2 can be written as a Chern-Simons action based on supersymmetric extensions of $sl(2, R)$ [30]. Since higher spin theories containing only bosonic fields are based on the $sl(N, R)$ with $N > 2$ [31, 32], it is natural to look for supersymmetric extensions of the $sl(N, R)$ algebra to construct consistent interacting higher spin theories in AdS_3 containing fermions. Given any such superalgebra \mathcal{G} the parity invariant Chern-Simons action is given by

$$S = \frac{k}{2\pi} \int \left[\text{str} \left(\Gamma d\Gamma + \frac{2}{3} \Gamma^3 \right) - \text{str} \left(\tilde{\Gamma} d\tilde{\Gamma} + \frac{2}{3} \tilde{\Gamma}^3 \right) \right]. \quad (2.1)$$

Here $\Gamma, \tilde{\Gamma}$ are the 1-forms which take values in \mathcal{G} and str refers to the super-trace over the respective algebras. The integral is over the 3 dimension space time. The equations of motion of this action are the following flatness conditions

$$d\Gamma + \Gamma \wedge \Gamma = 0, \quad d\tilde{\Gamma} + \tilde{\Gamma} \wedge \tilde{\Gamma} = 0. \quad (2.2)$$

To obtain the equations of motion in component form, one needs to expand $\Gamma, \tilde{\Gamma}$ in terms of the generators of the superalgebra. The coefficients of this expansion are the

fields of the theory, this is then substituted in the equations given in (2.2) to obtain the equations of motion in the component form. Thus to write down the equations of motion it is sufficient to know the structure of the algebra.

The generalized Killing spinor equations

It is also easy to write down the Killing spinor equations. Let the bosonic generators of the algebra be denoted by T_a and the corresponding bosonic fields by A_a . Similarly let the fermionic generators be G_i and. Consider a bosonic solution to the equations of motion. Then one has the following equation

$$d(A^a T_a) + (A^a T_a) \wedge (A^b T_b) = 0. \quad (2.3)$$

Note that in this language the bosonic fields A^a are 1-forms. The Killing spinor equation is essentially the equation that demands that the background ϕ^a is invariant under fermionic gauge transformations. Let ϵ^i be the parameters of this transformation, then the equation is given by

$$\delta\psi \equiv \partial_\mu \epsilon^i G_i + A_\mu^a \epsilon^i [T_a, G_i] = 0. \quad (2.4)$$

This is essentially the equation demanding that the covariant derivative in presence of the bosonic background A_μ^a vanishes. The solutions ϵ^i are the Killing spinors. It is clear that the variation $\delta\psi$ is a fermionic symmetry of the Lagrangian. By demanding $\delta\psi = 0$ we are looking for general variations with parameters involving fermions both with spin 1/2 and 3/2 which leaves the background invariant. This is the generalized notion of the Killing spinor in the higher spin theory. It is important to note that flatness conditions in (2.3) are the integrability constraints of the Killing spinor equation (2.4). Thus given that a bosonic background satisfies the equations of motion solutions to the Killing spinor equations are guaranteed to exist. However we must also impose the condition that the Killing spinors are periodic with respect to the angular co-ordinate in AdS_3 . This then decides the condition whether a given background is supersymmetric.

The class of superalgebras we will be interested in belongs to $sl(N|N-1)$. However the general conclusion regarding the supersymmetry of a given background drawn at the end of our analysis applies to any superalgebra. An appropriate basis to discuss the $sl(N|N-1)$ algebra is the explicit matrix representation of the algebra given in section 61 of [33]. This is in the Cartan-Weyl basis which is suitable for the general analysis of the Killing spinor and the supersymmetric conditions. For most parts of the paper we will explicitly study the case of $sl(3|2)$. The bosonic part of this algebra is given by $sl(3) \oplus sl(2) \oplus u(1)$. This algebra contains $sl(2|1)$ on which (2,2) supergravity in AdS_3 is based.

2.1 The $sl(3|2)$ superalgebra

In this section we explicitly write down the commutation relations of $sl(3|2)$. We obtain this by taking the large central charge and the global part of the $\mathcal{N} = 2$ super \mathcal{W}_3 algebra written down by [27]. This provides evidence that the boundary theory of Chern-Simons gravity based on $sl(3|2)$ is a super conformal theory with $\mathcal{N} = 2$ super \mathcal{W}_3 symmetry.

$\mathcal{N} = 2$ super \mathcal{W}_3 super conformal algebra contains generators with J, G^\pm, L with spin 1, 3/2 and 2 respectively. These generators obey the $\mathcal{N} = (2, 2)$ superconformal algebra among themselves. J is the generator of the R-symmetry, G^\pm are the supersymmetry generators and L is the stress tensor. In addition to this there is also the generators V, U, W with spin 2, 5/2, 3 respectively. W generates the super conformal \mathcal{W}_3 symmetry. Taking the large central charge limit and the global part of the commutation relations of $\mathcal{N} = 2$ super conformal \mathcal{W}_3 we obtain the following:

$$\begin{aligned}
[J, J] &= 0, & [L_m, L_n] &= (m - n)L_{m+n}, \\
[V_m, V_n] &= (m - n)(L_{m+n} + \kappa V_{m+n}), \\
[W_m, W_n] &= \frac{1}{4}(m - n)(2m^2 + 2n^2 - mn - 8)(L_{m+n} + \frac{\kappa}{5}V_{m+n}), \\
[J, L_n] &= 0, & [J, V_n] &= 0, & [J, W_n] &= 0, \\
[L_m, V_n] &= (m - n)V_{m+n}, & [L_m, W_n] &= (2m - n)W_{m+n}, \\
[V_m, W_n] &= \frac{\kappa}{5}(2m - n)W_{m+n}.
\end{aligned} \tag{2.5}$$

Here the subscripts m, n on the generators L run from $-1, 0, 1$ while the subscripts on the generators W run from $-2, -1, 0, 1, 2$. The commutation relations between bosonic and fermionic generators are given by

$$\begin{aligned}
[L_m, G_r^\pm] &= (\frac{1}{2}m - r)G_{m+r}^\pm, & [J, G_r^\pm] &= \pm G_r^\pm, \\
[L_m, U_r^\pm] &= (\frac{3}{2}m - r)U_{m+r}^\pm, & [J, U_r^\pm] &= \pm U_r^\pm, \\
[V_m, G_r^\pm] &= \pm U_{r+m}^\pm, & [G_r^\pm, W_m] &= (2r - \frac{1}{2}m)U_{r+m}^\pm, \\
[V_m, U_r^+] &= \frac{2}{5}\kappa(\frac{3}{2}m - r)U_{m+r}^+ + \frac{1}{4}(3m^2 - 2mr + r^2 - \frac{9}{4})G_{m+r}^+, \\
[V_m, U_r^-] &= -\frac{2}{5}\kappa^*(\frac{3}{2}m - r)U_{m+r}^- - \frac{1}{4}(3m^2 - 2mr + r^2 - \frac{9}{4})G_{m+r}^-, \\
[U_r^+, W_m] &= \frac{\kappa}{10}(2r^2 - 2rm + m^2 - \frac{5}{2})U_{r+m}^+ \\
&\quad + \frac{1}{8}(4r^3 - 3r^2m + 2rm^2 - m^3 - 9r + \frac{19}{4}m)G_{r+m}^+, \\
[U_r^-, W_m] &= \frac{\kappa^*}{10}(2r^2 - 2rm + m^2 - \frac{5}{2})U_{r+m}^- \\
&\quad + \frac{1}{8}(4r^3 - 3r^2m + 2rm^2 - m^3 - 9r + \frac{19}{4}m)G_{r+m}^-.
\end{aligned} \tag{2.6}$$

Here the subscripts r, s on G^\pm run from $-1/2, 1/2$ while the subscripts on the generators U^\pm run from $-3/2, -1/2, 1/2, 3/2$. Finally the anti-commutation rules between

the fermionic generators are given by

$$\begin{aligned}
\{G_r^\pm, G_s^\mp\} &= 2L_{r+s} \pm (r-s)J, & \{G_r^\pm, G_s^\pm\} &= 0, \\
\{G_r^\pm, U_s^\mp\} &= 2W_{r+s} \pm (3r-s)V_{r+s}, & \{G_r^\pm, U_s^\pm\} &= 0, \\
\{U_r^+, U_s^-\} &= -\frac{2}{5}\kappa(r-s)W_{r+s} + (3s^2 - 4rs + 3r^2 - \frac{9}{2})(\frac{1}{2}L_{r+s} + \frac{\kappa}{5}V_{r+s}) \\
&\quad + \frac{1}{4}(r-s)(r^2 + s^2 - \frac{5}{2})J_{r+s}, \\
\{U_r^\pm, U_s^\pm\} &= 0.
\end{aligned} \tag{2.7}$$

On taking large central charge limit of the non-linear terms in the super \mathcal{W}_3 algebra drop off and we obtain where $\kappa = \pm(5/2)i$. We have verified that all the Jacobi identities of this algebra are satisfied using the Quantum add-on for Mathematica [34].

To see that the bosonic part of the algebra given in (2.5) is given by the direct sum $sl(3) \oplus sl(2) \oplus u(1)$, we define the following linear combinations of generators

$$T_m^+ = -\frac{1}{3}(L_m + 2iV_m) \quad T_m^- = \frac{1}{3}(4L_m + 2iV_m). \tag{2.8}$$

Substituting these redefinitions in (2.5) we obtain

$$[T_m^+, T_n^-] = 0, \quad [T_m^+, W_n] = 0, \tag{2.9}$$

and the can show that the generators T_m^+ obey the $sl(2)$ algebra while the generators T_n^-, W_m obey the commutation relations of the $sl(3)$ algebra given by

$$\begin{aligned}
[T_m^-, T_n^-] &= (m-n)T_{m+n}^-, & [T_m^-, W_n] &= (2m-n)W_{m+n}, \\
[W_m, W_n] &= \frac{3}{16}(m-n)(2m^2 + 2n^2 - mn - 8)T_{m+n}^-.
\end{aligned} \tag{2.10}$$

Note the comparing the $sl(3)$ algebra given in equation (A.2) of [18] we see that the parameter σ defined in those equations is equal to $(3/4)^2$. Now that we have the explicit $sl(3|2)$ algebra we can proceed to obtain solutions to the equations of motion and study their supersymmetry. The traces of the product of any two of the $sl(3)$ generators is the same as that of equation (A.3) of [18] with $\sigma = (3/4)^2$, while for the $sl(2)$ we use the the representation in terms of the Pauli matrices.

3. Supersymmetry of classical solutions

In this section we begin by describing the general strategy we adopt to find the Killing spinors for the various backgrounds considered in this paper. We reduce the Killing spinor equation to essentially a set of ordinary first order equations with constant coefficients which can then be easily solved. In section 3.2 we construct the general higher spin conical defects in the $sl(3|2)$ theory. These solutions in general have fields

in the $sl(3) \oplus sl(2) \oplus u(1)$ directions. We then solve the Killing spinor equations and determine the supersymmetric conditions for the supercharges with $u(1)$ charge in one copy of $sl(3|2)$ in the Chern-Simons theory. This analysis can be generalized for the remaining charges. We also determine special points in the parameter space of conical defects which reduce to AdS_3 . In section 3.3 we study the supersymmetry of black holes in this theory. This includes the usual BTZ black hole embedded in $sl(2)$, the higher spin black hole of [18] embedded in $sl(3)$ along with the $u(1)$ turned on. We also construct a new black hole solution which has charges in $sl(3) \oplus sl(2) \oplus u(1)$ and study its supersymmetry.

3.1 General strategy to obtain the Killing spinors

The gauge connections which will be of interest in this paper has the generic form as follows

$$A = \left(\sum_{m=-1}^1 (t_m e^{m\rho} T_m^- + s_m e^{m\rho} T_m^+) + \sum_{m=-2}^2 (w_m e^{m\rho} W_m) + \xi J \right) dx^+ - \xi J dx^- + (T_0^+ + T_0^-) d\rho. \quad (3.1)$$

where $x^\pm = t \pm \phi$ and ρ, t, ϕ are the radial, time and the angular co-ordinates of the three dimensional spaces we consider. The connection in (3.1) obeys the flatness condition. The general form of the connection can be conveniently written as $A = a_\mu^n e^{(n)\rho} T_n$. T_n being a bosonic generator of the superalgebra. Negative weights of the generators with respect to $T_0^+ + T_0^- (= L_0)$ appear in the exponential factors. For example, we have terms like $w_+^{(2)} e^{2\rho} W_2$ and $t_+^{(-1)} e^{-\rho} T_{-1}^+$.

The Killing spinor is given by

$$(\partial_\mu \epsilon^r) G_r + \epsilon^a A_\mu^b [T_b, G_a] = 0. \quad (3.2)$$

$[T_b, G_a]$ is again some linear combination of the fermionic generators which we can write as $f_{bac} G_c$. Here f_{bac} are the structure constants of the superalgebra and b is a bosonic index while a and c are fermionic ones. Substituting for the commutation relation in (3.2) we obtain the following equation

$$(\partial_\mu \epsilon^r) G_r + \epsilon^a A_\mu^b f_{bac} G_c = 0, \quad (3.3)$$

Now defining the matrix $(\mathbb{M}_\mu)_{ac} = A_\mu^b f_{bac}$ we finally arrive at the matrix equation.

$$\partial_\mu \epsilon^c + (\mathbb{M}_\mu)_a^c \epsilon^a = 0. \quad (3.4)$$

Our task now is to solve (3.4). In order to do this we make the following ansatz for the solution.

$$\epsilon = \mathcal{R}(\rho) e^{\xi x^-} f(x_+), \quad (3.5)$$

where $\mathcal{R}(\rho)$ is a square matrix which is given by

$$\mathcal{R}(\rho) = \begin{pmatrix} e^{-\rho/2} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\rho/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-3\rho/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\rho/2} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\rho/2} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{3\rho/2} \end{pmatrix}. \quad (3.6)$$

This ansatz solves the ρ dependence because the matrix \mathbb{M}_ρ has the form $\text{Diag}(1/2, -1/2, 3/2, 1/2, -1/2, -3/2)$.

For connections of the type (3.1) we have the following property:

$$\mathcal{R}^{-1}(\rho)\mathbb{M}_\mu\mathcal{R}(\rho) \text{ is independent of } \rho \quad (3.7)$$

This can be seen by considering the definitions of \mathbb{M}_μ and $\mathcal{R}(\rho)$. We have

$$\begin{aligned} \mathcal{R}_{ea}^{-1}(\mathbb{M}_\mu)_{ac}\mathcal{R}_{cd} &= (e^{-(a)\rho}\delta_{ea}) (f_{bac}A_\mu^b) (e^{(c)\rho}\delta_{cd}), \\ &= (e^{-(a)\rho}\delta_{ea}) (f_{bac}a_\mu^b e^{(b)\rho}) (e^{(c)\rho}\delta_{cd}), \\ &= e^{-(a+b-c)\rho}\delta_{ea}\delta_{cd}f_{bac}a_\mu^b. \end{aligned} \quad (3.8)$$

As mentioned earlier, negative weights of the corresponding generators appear in the exponential factors. Note that we have $[T_b, G_a] \sim G_{a+b}$, therefore f_{bac} is non-zero only when $a + b = c$. This removes the ρ dependence from (3.8) and proves (3.7). We then have

$$\mathcal{R}_{ea}^{-1}(\mathbb{M}_\mu)_{ac}\mathcal{R}_{cd} = f_{bed}a_\mu^b. \quad (3.9)$$

$f(x_+)$ in (3.5) is a column vector which solves the x_+ dependence of the Killing spinor. Using the property (3.7) in the $+$ component of the Killing spinor equation (3.4) we obtain

$$\partial_+ f(x_+) + [\mathcal{R}^{-1}(\mathbb{M}_+) \mathcal{R}] f(x_+) = 0. \quad (3.10)$$

Now let λ_i be the eigenvalues of the constant matrix $\mathcal{R}^{-1}(\mathbb{M}_+)\mathcal{R}$, then the solution for the above equation is

$$f(x_+) = \sum_i c_i e^{-\lambda_i x_+} \mathbf{z}_i, \quad (3.11)$$

where \mathbf{z}_i is the eigenvector of $\mathcal{R}^{-1}(\mathbb{M}_+)\mathcal{R}$ corresponding to the eigenvalue λ_i .

Finally the x_- dependence of the Killing spinor is captured by the simple factor $e^{\xi x_-}$. This is due to the fact that $(\mathbb{M}_-)_{cd} = -\xi\delta_{cd}$.

3.2 Conical defects

Metric and gauge connections

We shall generalize the solution of [19] to include the spin-1 gauge field corresponding to the generators J and the additional spin-2 field corresponding to the generators

V. We start with the 1-forms, written in terms of the decoupled generators, T^+ and T^- as defined in (2.8)

$$A = (e^{-\rho}\delta_{-1}T_{-1}^+ + e^{\rho}\delta_1T_1^+ + e^{-\rho}\beta_{-1}T_{-1}^- + e^{\rho}\beta_1T_1^- + e^{-\rho}\eta_{-1}W_{-1} + e^{\rho}\eta_1W_1 + \xi J)dx^+ - \xi Jdx^- + (T_0^- + T_0^+)d\rho, \quad (3.12)$$

$$\bar{A} = -(e^{-\rho}\bar{\delta}_{-1}T_{-1}^+ + e^{\rho}\bar{\delta}_1T_1^+ + e^{-\rho}\bar{\beta}_{-1}T_{-1}^- + e^{\rho}\bar{\beta}_1T_1^- + e^{-\rho}\bar{\eta}_{-1}W_{-1} + e^{\rho}\bar{\eta}_1W_1 - \xi J)dx^- - \xi Jdx^+ - (T_0^- + T_0^+)d\rho. \quad (3.13)$$

Note that here we have chosen the same notations to label the generators in the second copy of $sl(3|2)$. Since, A and \bar{A} are linear combinations of the tetrad (e) and the vielbein (ω) [31, 32], we can extract them from the above. The non-zero components of the tetrad turn out to be

$$\begin{aligned} e_{\rho} &= L_0, \\ e_+ &= \frac{1}{2}(e^{-\rho}\delta_{-1}T_{-1}^+ + e^{\rho}\delta_1T_1^+ + e^{-\rho}\beta_{-1}T_{-1}^- + e^{\rho}\beta_1T_1^- + e^{-\rho}\eta_{-1}W_{-1} + e^{\rho}\eta_1W_1), \\ e_- &= \frac{1}{2}(e^{-\rho}\bar{\delta}_{-1}T_{-1}^+ + e^{\rho}\bar{\delta}_1T_1^+ + e^{-\rho}\bar{\beta}_{-1}T_{-1}^- + e^{\rho}\bar{\beta}_1T_1^- + e^{-\rho}\bar{\eta}_{-1}W_{-1} + e^{\rho}\bar{\eta}_1W_1). \end{aligned} \quad (3.14)$$

The metric is given by the following formula.

$$g_{\mu\nu} = \frac{1}{\epsilon_{(3|2)}} \text{str}(e_{\mu}e_{\nu}), \quad (3.15)$$

where $\epsilon_{(3|2)} = \text{str}(L_0^2) = \text{str}(T_0^+ + T_0^-)^2$. Evaluating it explicitly we obtain $\epsilon_{(3|2)} = 3/4$. By choosing this normalization we have chosen the gravitational $sl(2)$ to be the those corresponding to the generators L_{\pm}, L_0 . From the commutation relations in (2.5) and (2.6) we see that it is under these generators that all fields have well defined weights. From the super \mathcal{W}_3 conformal field theory point of view these are the modes which are part of the stress tensor of the theory. One then obtains

$$\begin{aligned} g_{\rho\rho} &= 1, \\ g_{++} &= -\frac{2}{3}(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}), \\ g_{--} &= -\frac{2}{3}(\bar{\beta}_1\bar{\beta}_{-1} - \frac{9}{16}\bar{\eta}_1\bar{\eta}_{-1} - \frac{1}{4}\bar{\delta}_1\bar{\delta}_{-1}). \end{aligned} \quad (3.16)$$

We now demand $g_{++} = g_{--}$. This results in the following equations

$$\bar{\delta}_{\pm 1} = \zeta^{\pm 1}\delta_{\pm 1}, \quad \bar{\beta}_{\pm 1} = \zeta^{\pm 1}\beta_{\pm 1}, \quad \bar{\eta}_{\pm 1} = \zeta^{\pm 1}\eta_{\pm 1}. \quad (3.17)$$

where ζ is constant. g_{++} and g_{--} now become

$$g_{\pm\pm} = -\frac{2}{3}(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}), \quad (3.18)$$

and g_{+-} has the form

$$g_{+-} = \frac{2}{3} \left(-\frac{1}{\zeta} (\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{4}\delta_{-1}^2) e^{-2\rho} - \zeta (\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{4}\delta_1^2) e^{2\rho} \right). \quad (3.19)$$

The metric then in terms of the (ρ, t, ϕ) coordinates is as follows.

$$\begin{aligned}
ds^2 = & d\rho^2 \\
& - \frac{4}{3} \left(\zeta(\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{4}\delta_1^2)e^{2\rho} + 2(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}) \right. \\
& \quad \left. + \zeta^{-1}(\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{4}\delta_{-1}^2)e^{-2\rho} \right) dt^2, \\
& + \frac{4}{3} \left(\zeta(\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{4}\delta_1^2)e^{2\rho} - 2(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}) \right. \\
& \quad \left. + \zeta^{-1}(\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{4}\delta_{-1}^2)e^{-2\rho} \right) d\phi^2.
\end{aligned}$$

We now need to impose the fact that g_{tt} and $g_{\phi\phi}$ need to have a $(\dots)^2$ form. The results in the following equation

$$(\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{2}\delta_1^2)(\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{2}\delta_{-1}^2) = (\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1})^2 \quad (3.20)$$

This imposes the conditions

$$\delta_{-1} = \alpha\delta_1, \quad \beta_{-1} = \alpha\beta_1, \quad \eta_{-1} = \alpha\eta_1. \quad (3.21)$$

Defining $\delta = \delta_1$, $\beta = \beta_1$ and $\eta = \eta_1$, the final form the metric with these conditions is

$$ds^2 = d\rho^2 - \frac{4}{3}(\beta^2 - (\frac{3}{4}\eta)^2 - (\frac{1}{2}\delta)^2) \left[\left(\sqrt{\zeta}e^\rho + \frac{\alpha}{\sqrt{\zeta}}e^{-\rho} \right)^2 dt^2 - \left(\sqrt{\zeta}e^\rho - \frac{\alpha}{\sqrt{\zeta}}e^{-\rho} \right)^2 d\phi^2 \right]. \quad (3.22)$$

By redefining ρ as $\rho \rightarrow \rho - \frac{1}{2} \log(\frac{\zeta}{\alpha})$ we can write (3.22) as

$$ds^2 = d\rho^2 - \frac{16\alpha}{3}(\beta^2 - (\frac{3}{4}\eta)^2 - (\frac{1}{2}\delta)^2) [(\sinh^2 \rho)dt^2 - (\cosh^2 \rho)d\phi^2]. \quad (3.23)$$

Killing spinors for the higher spin conical defect

The equation for the covariantly conserved spinor (2.4) is

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.24)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.25)$$

From the analysis of the previous section the gauge connection for the higher conical defect is given by

$$\begin{aligned}
A = & (\alpha\delta e^{-\rho}T_{-1}^+ + \delta e^\rho T_1^+ + \alpha\beta e^{-\rho}T_1^- + \beta e^\rho T_{-1}^- + \alpha\eta e^{-\rho}W_{-1} + \eta e^\rho W_1 + \xi J)dx^+ \\
& - \xi J_0 dx^- + L_0 d\rho.
\end{aligned} \quad (3.26)$$

where $L_0 = T_0^+ + T_0^-$. We will study the supersymmetry of only one copy of the $(sl(N) \oplus sl(N-1) \oplus u(1))_L \times (sl(N) \oplus sl(N-1) \oplus u(1))_R$ Chern-Simons theory. A similar analysis can be repeated for the second copy.

Writing out the components as in (3.1) (but with the exponential factors) of this connection explicitly we have

$$\begin{aligned} j_+ &= \xi, & j_- &= -\xi \\ s_+^{-1} &= \alpha \delta e^{-\rho}, & s_+^1 &= \delta e^\rho, & l_\rho^0 &= 1, \\ t_+^{-1} &= \alpha \beta e^{-\rho}, & t_+^1 &= \beta e^\rho, \\ w_+^{-1} &= \alpha \eta e^{-\rho}, & w_+^1 &= \eta e^\rho, \end{aligned} \quad (3.27)$$

where, t and s are the components corresponding to the generators T^+ and T^- respectively. Written explicitly, the equation (3.24) for the components G_r^+ and U_r^+ is given by

$$\begin{aligned} & \partial_\mu \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} \\ & + \frac{1}{4} \begin{pmatrix} 4j_0 + 2l_0 & \frac{16}{3}s_{-1} + \frac{4}{3}t_{-1} & 4i(s_{-1} - t_{-1}) + 3w_{-1} & 0 & \frac{4i}{3}(s_{-1} - t_{-1}) + 3w_{-1} & 0 \\ \frac{16}{3}s_{-1} - \frac{4}{3}t_{-1} & 4j_0 - 2l_0 & 0 & \frac{4i}{3}(s_{-1} - t_{-1}) - 3w_{-1} & 0 & 4i(s_{-1} - t_{-1}) - 3w_{-1} \\ \frac{8i}{3}(s_{-1} - t_{-1}) + 2w_{-1} & 0 & 4j_0 + 6l_0 & -\frac{8}{3}s_{-1} - \frac{4}{3}t_{-1} + 2iw_{-1} & 0 & 0 \\ 0 & \frac{8i}{3}(s_{-1} - t_{-1}) - 6w_{-1} & 8s_{-1} + 4t_{-1} - 6iw_{-1} & 4j_0 + 2l_0 & -\frac{16}{3}s_{-1} - \frac{8}{3}t_{-1} & 0 \\ \frac{8i}{3}(s_{-1} - t_{-1}) + 6w_{-1} & 0 & 0 & \frac{16}{3}s_{-1} + \frac{8}{3}t_{-1} & 4j_0 - 2l_0 & -8s_{-1} - 4t_{-1} - 6iw_{-1} \\ 0 & \frac{8i}{3}(s_{-1} - t_{-1}) - 2w_{-1} & 0 & 0 & \frac{8}{3}s_{-1} + \frac{4}{3}t_{-1} + 2iw_{-1} & 4j_0 - 6l_0 \end{pmatrix}_\mu \\ & \times \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \end{aligned} \quad (3.28)$$

The x^+ dependence of the column spinor above is determined by the eigenvalues of the $\mathcal{R}^{-1}(\rho)\mathbb{M}_+\mathcal{R}(\rho)$ matrix. The solutions are of the form

$$\begin{aligned} & \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} \\ & = \mathcal{R}(\rho) e^{\xi(x_- - x_+)} (d_1 e^{i\sqrt{\alpha}\delta x_+ \mathbf{z}} + d_2 e^{-i\sqrt{\alpha}\delta x_+ \mathbf{z}_2} \\ & \quad + d_3 e^{i\sqrt{\alpha}\left(\delta + 2\left(\beta^2 - \left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+ \mathbf{z}_3} + d_4 e^{-i\sqrt{\alpha}\left(\delta + 2\left(\beta^2 - \left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+ \mathbf{z}_4} \\ & \quad + d_5 e^{i\sqrt{\alpha}\left(\delta - 2\left(\beta^2 - \left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+ \mathbf{z}_5} + d_6 e^{-i\sqrt{\alpha}\left(\delta - 2\left(\beta^2 - \left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+ \mathbf{z}_6}). \end{aligned} \quad (3.29)$$

The matrix \mathcal{R} has the ρ dependence as in (3.6).

The preceding discussion allows us to fix the value of ξ in so that we can impose proper periodicity conditions on the Killing spinor. The possibilities are the following:

$$2\xi = \pm i\sqrt{\alpha}\delta + in, \quad (3.30)$$

$$2\xi = \pm i\sqrt{\alpha} \left(\delta \pm 2(\beta^2 - (\frac{3}{4}\eta)^2)^{1/2} \right) + in. \quad (3.31)$$

where n is any integer.

One can also examine the Killing spinor equation for the G^- and U^- components. On performing the same analysis as above it is found that the component $u(1)$ gauge field, ξ has to be complex in order to impose proper periodicity requirements. Since this is not allowed, there are no Killing spinors corresponding to conjugates of the G^-, U^- charges.

We will now show that at special values of the parameter space, the general higher spin conical defect reduces to well known solutions.

Supersymmetry of conical defects in $sl(2)$

Embedding the conical defect solution only in the $sl(2) \oplus u(1)$ sub-algebra we have the following gauge connections

$$A = \left(e^\rho T_1^+ + \frac{\gamma}{4} e^{-\rho} T_{-1}^+ \right) dx^+ + T_0^+ d\rho + 2\xi J d\phi, \quad (3.32)$$

$$\bar{A} = - \left(e^\rho T_{-1}^+ + \frac{\gamma}{4} e^{-\rho} T_1^+ \right) dx^+ - T_0^+ d\rho + 2\xi J d\phi.$$

Note that this gauge connection is a special case of the higher spin conical defect with $\alpha = \gamma/4$, $\delta = 1$ and $\beta = \eta = 0$.

One can perform a gauge transformation $A \rightarrow U^-(A + d)U$ with $U = e^{\rho T_0^-}$ on the connection (3.32). The new connections are then of the form

$$A = \left(e^\rho T_1^+ + \frac{\gamma}{4} e^{-\rho} T_{-1}^+ \right) dx^+ + (T_0^+ + T_0^-) d\rho + 2\xi J d\phi, \quad (3.33)$$

$$\bar{A} = - \left(e^\rho T_{-1}^+ + \frac{\gamma}{4} e^{-\rho} T_1^+ \right) dx^+ - (T_0^+ + T_0^-) d\rho + 2\xi J d\phi.$$

where, for \bar{A} we have used the transformation by $U = e^{-\rho T_0^-}$. Now the gauge connections are of the general form given in (3.1).

The equation for the covariantly constant spinor is

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.34)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.35)$$

The analysis for the Killing spinor performed for the case of the higher spin conical defect can be repeated. The solutions of the components of the generators G^\pm, U^\pm are of the form

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix}_\pm = \mathcal{R}(\rho) e^{\xi(x_- - x_+)} \left(e^{i\frac{\sqrt{\gamma}}{2}x_+} (d_1 \mathbf{z}_1 + d_2 \mathbf{z}_2 + d_3 \mathbf{z}_3) + e^{-i\frac{\sqrt{\gamma}}{2}x_+} (d_4 \mathbf{z}_4 + d_5 \mathbf{z}_5 + d_6 \mathbf{z}_6) \right)_\pm \quad (3.36)$$

\mathbf{z}_i are the eigenvectors of the 6×6 matrices which appear in the Killing spinor equation. The ρ dependence is contained in the matrix $\mathcal{R}(\rho)$ given in (3.6).

In order to impose proper periodicity requirements on our Killing spinor, we need to set,

$$\xi = \pm i \frac{\sqrt{\gamma}}{4} + in. \quad (3.37)$$

Note that this condition coincides with the condition found for Killing spinors in [23]. Since there is a pair of eigen values with degeneracy 3, we will in general have 3 Killing spinors which will satisfy the periodicity condition.

Supersymmetry of Anti-deSitter space in $sl(2)$

For the case of AdS_3 one can perform the same analysis with $\gamma = 1$. As expected, it can be seen that one does not require the $u(1)$ gauge field and one can obtain anti-periodic Killing spinors. The solution for the Killing spinors for this case is

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix}_\pm = \mathcal{R}(\rho) \left(e^{i\frac{x_\pm}{2}} (d_1 \mathbf{z}_1 + d_2 \mathbf{z}_2 + d_3 \mathbf{z}_3) + e^{-i\frac{x_\pm}{2}} (d_4 \mathbf{z}_4 + d_5 \mathbf{z}_5 + d_6 \mathbf{z}_6) \right)_\pm. \quad (3.38)$$

The ρ dependence of the Killing spinor remains the same as the one for the conical defect.

Supersymmetry of conical defects in the gravitational $sl(2)$

The metric for the conical defect embedded in the gravitational $sl(2)$, which is generated by the L_m generators written in Fefferman-Graham coordinates is

$$ds^2 = d\rho^2 - \left(e^\rho + \frac{\gamma}{4} e^{-\rho} \right)^2 dt^2 + \left(e^\rho - \frac{\gamma}{4} e^{-\rho} \right)^2 d\phi^2. \quad (3.39)$$

This can be equivalently written in terms of the gauge connections

$$A = \left(e^\rho L_1 + \frac{\gamma}{4} e^{-\rho} L_{-1} \right) dx^+ + L_0 d\rho + 2\xi J_0 d\phi, \quad (3.40)$$

$$\bar{A} = - \left(e^\rho L_{-1} + \frac{\gamma}{4} e^{-\rho} L_1 \right) dx^+ - L_0 d\rho + 2\xi J_0 d\phi. \quad (3.41)$$

Note that this connection is the special case of (3.26) with $\beta = \delta$ and $\eta = 0$. These connections and the metric reduce to that of global AdS by setting $\gamma = 1$ and $\xi = 0$. The connections in terms of the components are

$$l_+^1 = e^\rho, \quad l_+^{-1} = \frac{\gamma}{4} e^{-\rho}, \quad l_\rho^0 = 1, \quad j_+^0 = \xi, \quad j_-^0 = -\xi. \quad (3.42)$$

the other components are zero.

The equation for the covariantly conserved spinor is

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.43)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.44)$$

For this case of the gauge connection (3.40) the Killing spinor equations for the G_r^\pm and U_r^\pm decouple. The equations for the $G_{\pm 1/2}^\pm$ components can be written as matrix equations as follows

$$\partial_\mu \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(2j^0 + l^0)_\mu & -l_\mu^{-1} \\ l_\mu^1 & \frac{1}{2}(2j^0 - l^0)_\mu \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} = 0. \quad (3.45)$$

The solutions are given by

$$\begin{aligned} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} &= \mathcal{R}(\rho) e^{\xi(x_- - x_+)} (c_1 e^{i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{y}_1 + c_2 e^{-i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{y}_2) \\ &= \mathcal{R}(\rho) e^{-2\xi\phi} (c_1 e^{i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{y}_1 + c_2 e^{-i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{y}_2), \end{aligned} \quad (3.46)$$

where, $\mathbf{y}_{1,2}$ are the eigenvectors of the matrix $\mathcal{R}^{-1} \mathbb{M}_+ \mathcal{R}$ (\mathbb{M}_μ being the matrix appearing in the equation (3.45)) and $\mathcal{R}(\rho)$ is a diagonal matrix having the ρ dependence

$$\mathcal{R}(\rho) = \begin{pmatrix} e^{-\rho/2} & 0 \\ 0 & e^{\rho/2} \end{pmatrix}. \quad (3.47)$$

The equations for the U_r^+ generators can be written as

$$\partial_\mu \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(2j^0 + 3l^0)_\mu & -l_\mu^{-1} & 0 & 0 \\ 3l_\mu^1 & \frac{1}{2}(2j^0 + l^0)_\mu & -2l_\mu^{-1} & 0 \\ 0 & 2l_\mu^1 & \frac{1}{2}(2j^0 - l^0)_\mu & -3l_\mu^{-1} \\ 0 & 0 & l_\mu^1 & \frac{1}{2}(2j^0 - 3l^0)_\mu \end{pmatrix} \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.48)$$

The solutions are given by

$$\begin{aligned}
& \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} \\
&= \mathcal{R}_2(\rho) e^{\xi(x_- - x_+)} (d_1 e^{i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{z}_1 + d_2 e^{-i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{z}_2 \\
&\quad + d_3 e^{i\frac{3\sqrt{\gamma}}{2}x_+} \mathbf{z}_3 + d_4 e^{-i\frac{3\sqrt{\gamma}}{2}x_+} \mathbf{z}_4), \\
&= \mathcal{R}_2(\rho) e^{-2\xi\phi} (d_1 e^{i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_1 + d_2 e^{-i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_2 \\
&\quad + d_3 e^{i\frac{3\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_3 + d_4 e^{-i\frac{3\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_4). \tag{3.49}
\end{aligned}$$

The matrix \mathcal{R}_2 has the ρ dependence

$$\mathcal{R}_2(\rho) = \begin{pmatrix} e^{-3\rho/2} & 0 & 0 & 0 \\ 0 & e^{-\rho/2} & 0 & 0 \\ 0 & 0 & e^{\rho/2} & 0 \\ 0 & 0 & 0 & e^{3\rho/2} \end{pmatrix}. \tag{3.50}$$

We thus get 6 independent Killing spinors. The condition which we get on demanding the proper periodicity of the spinor is

$$\xi = \pm i\frac{\gamma}{4} + in, \quad \text{or} \quad \xi = \pm 3i\frac{\gamma}{4} + in. \tag{3.51}$$

Thus, on embedding this conical defect in the $sl(2)$ corresponding to L_0, L_{\pm} we see that there are 4 eigenvalues out of which there are 2 doubly degenerate ones. In fact these two match with that given in (3.37) which also agrees with [23].

The Killing spinor equations for the G_r^- and U_r^- components of also form a set of 6 coupled equations. These equations are the same as the above with the replacement $j_0 \rightarrow -j_0$ or $\xi \rightarrow -(-\xi)$. Thus, they admit same solutions as given in (3.46) and (3.49) with different arbitrary constants

Supersymmetry of anti-de Sitter space in the gravitational $sl(2)$

Let us now turn to the case of global AdS_3 embedded in the gravitational $sl(2)$. This is a special case of the conical spaces embedded in the gravitational $sl(2)$ with $\gamma = 1, \xi = 0$. The metric in terms of the Fefferman-Graham coordinates is

$$ds^2 = d\rho^2 - \left(e^{\rho} + \frac{1}{4}e^{-\rho} \right)^2 dt^2 + \left(e^{\rho} - \frac{1}{4}e^{-\rho} \right)^2 d\phi^2. \tag{3.52}$$

This can be equivalently written in terms of the gauge connections

$$\begin{aligned} A &= \left(e^\rho L_1 + \frac{1}{4} e^{-\rho} L_{-1} \right) dx^+ + L_0 d\rho, \\ \bar{A} &= - \left(e^\rho L_{-1} + \frac{1}{4} e^{-\rho} L_1 \right) dx^+ - L_0 d\rho. \end{aligned} \quad (3.53)$$

The solutions are given by

$$\begin{aligned} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} &= \mathcal{R}(\rho) (c_1 e^{\frac{i}{2}x^+} y_1 + c_2 e^{-\frac{i}{2}x^+} y_2), \\ &= \mathcal{R}(\rho) (c_1 e^{\frac{i}{2}(t+\phi)} y_1 + c_2 e^{-\frac{i}{2}(t+\phi)} y_2), \end{aligned} \quad (3.54)$$

and

$$\begin{aligned} \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} &= \mathcal{R}_2(\rho) (d_1 e^{\frac{i}{2}x^+} \mathbf{z}_1 + d_2 e^{-\frac{i}{2}x^+} \mathbf{z}_2 + d_3 e^{\frac{3i}{2}x^+} \mathbf{z}_3 + d_4 e^{-\frac{3i}{2}x^+} \mathbf{z}_4), \\ &= \mathcal{R}_2(\rho) (d_1 e^{\frac{i}{2}(t+\phi)} \mathbf{z}_1 + d_2 e^{-\frac{i}{2}(t+\phi)} \mathbf{z}_2 + d_3 e^{\frac{3i}{2}(t+\phi)} \mathbf{z}_3 + d_4 e^{-\frac{3i}{2}(t+\phi)} \mathbf{z}_4). \end{aligned} \quad (3.55)$$

We obtain 6 independent Killing spinors which are anti-periodic corresponding to the G_r^+ and U_r^+ generators. Similarly performing the same analysis it is easy to see that one obtains 6 independent anti-periodic Killing spinors corresponding to the G_r^- and U_r^- generators.

The holonomy of global AdS_3 embedded in the gravitational $sl(2)$ can be shown to be trivial. Thus this background corresponds to the supersymmetric vacuum in the Neveu-Schwarz sector of the dual CFT.

3.3 The BTZ black hole

The BTZ black hole in $sl(2)$

We now examine the supersymmetry of the connection corresponding to that of the BTZ black hole embedded in the $sl(2)$ part of bosonic algebra $sl(3) \oplus sl(2) \oplus u(1)$. The connections are given by

$$\begin{aligned} A &= \left(e^\rho T_1^+ - \frac{2\pi}{k} \mathcal{L} e^{-\rho} T_{-1}^+ \right) dx^+ + T_0^+ d\rho, \\ \bar{A} &= - \left(e^\rho T_{-1}^+ - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} T_1^+ \right) dx^- - T_0^+ d\rho, \end{aligned} \quad (3.56)$$

where

$$\mathcal{L} = \frac{M - \hat{J}}{4\pi}, \quad \bar{\mathcal{L}} = \frac{M + \hat{J}}{4\pi} \quad (3.57)$$

We shall make a gauge transformation $A \rightarrow U^-(A + d)U$ to the above connections with $U = e^{\rho T_0^-}$ for A and $U = e^{-\rho T_0^-}$ for \bar{A} . This gives

$$\begin{aligned} A &= \left(e^{\rho} T_1^+ - \frac{2\pi}{k} \mathcal{L} e^{-\rho} T_{-1}^+ \right) dx^+ (T_0^+ + T_0^-) d\rho, \\ \bar{A} &= - \left(e^{\rho} T_{-1}^+ - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} T_1^+ \right) dx^- - (T_0^+ + T_0^-) d\rho. \end{aligned} \quad (3.58)$$

Now the connection is of the general form given by (3.1).

For the extremal case ($M = \hat{J}$) we have, $\mathcal{L} = 0$ and therefore the connection A becomes

$$A = e^{\rho} T_1^+ dx^+ + (T_0^+ + T_0^-) d\rho. \quad (3.59)$$

The equation for the Killing spinor is as follows

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.60)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.61)$$

Written explicitly in form of a matrix equation the (3.60) reads for the ρ component for the component of the generators G_r^+ and U_r^+ as

$$\partial_\rho \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.62)$$

and for the $+$ component

$$\partial_+ \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} 0 & 0 & -i & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & -\frac{i}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{2i}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & -\frac{2i}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.63)$$

The solutions are then of the form

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \begin{pmatrix} c_3 e^{-\rho/2 - i\pi/2} \\ \frac{c_2}{2} e^{\rho/2 - i\pi/2} \\ 0 \\ c_3 e^{-\rho/2} \\ c_2 e^{\rho/2} \\ c_1 e^{3\rho/2} \end{pmatrix}. \quad (3.64)$$

Thus there are 3 linearly independent Killing spinors corresponding to the supercharges with positive J charge for the extremal BTZ embedded in the $sl(3|2)$ theory.

The BTZ black hole in gravitational $sl(2)$

When 3-dimensional gravity is described by a Chern-Simons theory, the BTZ black hole is given by the connections

$$\begin{aligned} A &= \left(e^\rho L_1 - \frac{2\pi}{k} \mathcal{L} e^{-\rho} L_{-1} \right) dx^+ + L_0 d\rho, \\ \bar{A} &= - \left(e^\rho L_{-1} - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} L_1 \right) dx^- - L_0 d\rho, \end{aligned} \quad (3.65)$$

where

$$\mathcal{L} = \frac{M - \hat{J}}{4\pi}, \quad \bar{\mathcal{L}} = \frac{M + \hat{J}}{4\pi}. \quad (3.66)$$

For the extremal case ($M = \hat{J}$) we have, $\mathcal{L} = 0$ and therefore the connection A becomes

$$A = e^\rho L_1 dx^+ + L_0 d\rho. \quad (3.67)$$

Thus the fields conjugate to the generators are

$$l_\rho^0 = 1, \quad l_+^1 = e^\rho. \quad (3.68)$$

The equation for the covariantly conserved spinor is

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.69)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.70)$$

The equations for the G_r^+ generators can be written as

$$\partial_\mu \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(2j^0 + l^0)_\mu & -l_\mu^{-1} \\ -l_\mu^1 & \frac{1}{2}(2j^0 - l^0)_\mu \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} = 0 \quad (3.71)$$

while that for U^+ generators are

$$\partial_\mu \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{3}{2}l_\mu^0 & 0 & 0 & 0 \\ 3l_\mu^1 & \frac{3}{2}l_\mu^0 & 0 & 0 \\ 0 & 2l_\mu^1 & -\frac{3}{2}l_\mu^0 & 0 \\ 0 & 0 & l_\mu^1 & -\frac{3}{2}l_\mu^0 \end{pmatrix} \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0 \quad (3.72)$$

and similar equations for $\tilde{\epsilon}$ and $\tilde{\lambda}$ with the replacements $\epsilon \rightarrow \tilde{\epsilon}$ and $\lambda \rightarrow \tilde{\lambda}$. The solution of these which obeys the proper periodicity requirements given by

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} = \begin{pmatrix} 0 \\ C e^{\rho/2} \end{pmatrix}, \quad \begin{pmatrix} \tilde{\epsilon}^{-1/2} \\ \tilde{\epsilon}^{1/2} \end{pmatrix} = \begin{pmatrix} 0 \\ \tilde{C} e^{\rho/2} \end{pmatrix} \quad (3.73)$$

$$\begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ D e^{\rho/2} \end{pmatrix}, \quad \begin{pmatrix} \tilde{\lambda}^{-3/2} \\ \tilde{\lambda}^{-1/2} \\ \tilde{\lambda}^{1/2} \\ \tilde{\lambda}^{3/2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{D} e^{\rho/2} \end{pmatrix} \quad (3.74)$$

The solution (3.73) matches with the one given in [28].

3.4 Higher spin black holes

We shall consider black holes with spin-3 charge recently obtained in [18]. The connections are given as

$$\begin{aligned} A = & \left(e^{\rho} T_1^- - \frac{2\pi}{k} \mathcal{L} e^{-\rho} T_{-1}^- + \frac{\pi}{2k\sigma} \mathcal{W} e^{-2\rho} W_{-2} \right) dx^+ \\ & + \mu \left(e^{2\rho} W_{-2} - \frac{4\pi \mathcal{L}}{k} W_0 + \frac{4\pi^2 \mathcal{L}^2}{k^2} e^{-2\rho} W_2 + \frac{4\pi \mathcal{W}}{k} e^{-\rho} T_{-1}^- \right) dx^- + 2\xi J d\phi + L_0 d\rho, \end{aligned} \quad (3.75)$$

$$\begin{aligned} \bar{A} = & - \left(e^{\rho} T_1^- - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} T_{-1}^- + \frac{\pi}{2k\sigma} \bar{\mathcal{W}} e^{-2\rho} W_{-2} \right) dx^+ \\ & - \bar{\mu} \left(e^{2\rho} W_{-2} - \frac{4\pi \bar{\mathcal{L}}}{k} W_0 + \frac{4\pi^2 \bar{\mathcal{L}}^2}{k^2} e^{-2\rho} W_2 + \frac{4\pi \bar{\mathcal{W}}}{k} e^{-\rho} T_{-1}^- \right) dx^- + 2\xi J d\phi - L_0 d\rho, \end{aligned} \quad (3.76)$$

where $L_0 = T_0^+ + T_{-1}^+$ and $\sigma = (3/4)^2$. These differ from the connection of [18] by a gauge transformation $U = e^{\rho T_0^+}$ and also contains a gauge field in the $u(1)$.

We shall consider the supersymmetry of the black hole with $\mathcal{W} = 0$ and $\mu = 0$ but $\bar{\mathcal{W}} \neq 0$ and $\bar{\mu} \neq 0$. The equation for the Killing spinor is as follows

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.77)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.78)$$

Written explicitly in form of a matrix equation the (3.77) reads

$$\partial_\rho \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.79)$$

$$\partial_+ \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \xi & \frac{8\mathcal{L}\pi}{3k} & i & 0 & -\frac{2i\mathcal{L}\pi}{3k} & 0 \\ \frac{4}{3} & \xi & 0 & \frac{i}{3} & 0 & -\frac{2i\mathcal{L}\pi}{k} \\ -\frac{4i\mathcal{L}\pi}{3k} & 0 & \xi & \frac{4\mathcal{L}\pi}{3k} & 0 & 0 \\ 0 & -\frac{4i\mathcal{L}\pi}{3k} & 2 & \xi & \frac{8\mathcal{L}\pi}{3k} & 0 \\ \frac{2i}{3} & 0 & 0 & \frac{4}{3} & \xi & \frac{4\mathcal{L}\pi}{k} \\ 0 & \frac{2i}{3} & 0 & 0 & \frac{2}{3} & \xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.80)$$

$$\partial_- \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} -\xi & 0 & 0 & 0 & 0 & 0 \\ 0 & -\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & -\xi & 0 \\ 0 & 0 & 0 & 0 & 0 & -\xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.81)$$

The solutions to these equations are given as

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \mathcal{R}(\rho) f_+(x_+) f_-(x_-), \quad (3.82)$$

where, $\mathcal{R}(\rho)$ is given in (3.6) and

$$f_+(x_+) = e^{-(2\sqrt{\frac{2\pi\mathcal{L}}{k}} + \xi)x_+} (c_1 \mathbf{y}_1 + c_2 \mathbf{y}_2) + e^{-(2\sqrt{\frac{2\pi\mathcal{L}}{k}} + \xi)x_+} (c_3 \mathbf{y}_3 + c_4 \mathbf{y}_4) + e^{-\xi x_+} (c_4 \mathbf{y}_4 + c_5 \mathbf{y}_5). \quad (3.83)$$

\mathbf{y}_i are the eigenvectors of the matrix that appears in the + component of the Killing spinor equation. As usual the x_- dependence is

$$f_-(x_-) = e^{\xi x_-}. \quad (3.84)$$

The value of the $u(1)$ field for which we get the proper periodicity of the spinor is

$$\xi = \pm \sqrt{\frac{2\pi\mathcal{L}}{k}}, \quad \text{or} \quad \xi = i\frac{n}{2}. \quad (3.85)$$

From degeneracy of the eigen values in (3.83) we see that in general we can have two Killing spinors for a given ξ satisfying any one of the conditions in (3.85).

A new higher spin black hole with a BTZ embedding

We shall now try to generalize the gauge connection (3.75) by including terms which involve the $sl(2)$ corresponding to the T_{\pm}^-, T_0^- generators. This solution is same as the one given in (3.75) but with the $sl(2)$ connections of BTZ the black hole added to it. It may thus admit a notion of the horizon. The connection is given as follows and we have verified that it obeys the flatness conditions.

$$\begin{aligned} A = & \left(e^{\rho} T_1^- - \frac{2\pi}{k} \mathcal{L}_1 e^{-\rho} T_{-1}^- + \frac{\pi}{2k\sigma} \mathcal{W} e^{-2\rho} W_{-2} + e^{\rho} T_1^+ - \frac{2\pi}{k} \mathcal{L}_2 e^{-\rho} T_{-1}^+ \right) dx^+ \\ & + \mu \left(e^{2\rho} W_2 - \frac{4\pi \mathcal{L}_1}{k} W_0 + \frac{4\pi^2 \mathcal{L}_1^2}{k^2} e^{-2\rho} W_{-2} + \frac{4\pi \mathcal{W}}{k} e^{-\rho} T_{-1}^- \right) dx^- + 2\xi J d\phi, \\ & + (T_0^- + T_0^+) d\rho \end{aligned} \quad (3.86)$$

$$\begin{aligned} \bar{A} = & - \left(e^{\rho} T_1^- - \frac{2\pi}{k} \bar{\mathcal{L}}_1 e^{-\rho} T_{-1}^- + \frac{\pi}{2k\sigma} \bar{\mathcal{W}} e^{-2\rho} W_{-2} + e^{\rho} T_1^+ - \frac{2\pi}{k} \bar{\mathcal{L}}_2 e^{-\rho} T_{-1}^+ \right) dx^+ \\ & - \bar{\mu} \left(e^{2\rho} W_2 - \frac{4\pi \bar{\mathcal{L}}_1}{k} W_0 + \frac{4\pi^2 \bar{\mathcal{L}}_1^2}{k^2} e^{-2\rho} W_{-2} + \frac{4\pi \bar{\mathcal{W}}}{k} e^{-\rho} T_{-1}^- \right) dx^- + 2\xi J d\phi, \\ & - (T_0^- + T_0^+) d\rho. \end{aligned} \quad (3.87)$$

with $\sigma = (3/4)^2$. The metric due to the above gauge connections is

$$\begin{aligned} ds^2 = & d\rho^2 - 3 \left(\mu e^{2\rho} dx^- + \frac{16\pi}{18k} \bar{\mathcal{W}} + \frac{4\pi^2}{k^2} \bar{\mu} \bar{\mathcal{L}}_1^2 e^{-2\rho} dx^+ \right) \\ & \times \left(\bar{\mu} e^{2\rho} dx^- + \frac{16\pi}{18k} \mathcal{W} + \frac{4\pi^2}{k^2} \mu \mathcal{L}_1^2 e^{-2\rho} dx^+ \right) \\ & - \frac{4}{3} \left(e^{\rho} dx^+ - \frac{2\pi}{k} \bar{\mathcal{L}}_1 e^{\rho} dx^- + \frac{4\pi}{k} \bar{\mu} \bar{\mathcal{W}} e^{-\rho} dx^+ \right) \left(e^{\rho} dx^+ - \frac{2\pi}{k} \mathcal{L}_1 e^{\rho} dx^- + \frac{4\pi}{k} \mu \mathcal{W} e^{-\rho} dx^+ \right) \\ & - \frac{1}{4} \left(\frac{4\pi}{k} \right)^2 (\mu \mathcal{L}_1 dx^- + \bar{\mu} \bar{\mathcal{L}}_1 dx^+)^2 - \frac{2\pi}{3k} (\mathcal{L}_2 (dx^+)^2 + \bar{\mathcal{L}}_2 (dx^-)^2) \\ & + \frac{1}{3} \left(e^{2\rho} + \left(\frac{2\pi}{k} \right)^2 \mathcal{L}_2 \bar{\mathcal{L}}_2 e^{-2\rho} \right) dx^+ dx^- \end{aligned} \quad (3.88)$$

We shall again consider the supersymmetry of the black hole with $\mathcal{W} = 0$ and $\mu = 0$. The equation for the Killing spinor is given by

$$\mathcal{D}_{\mu} \lambda \equiv \partial_{\mu} \lambda + [A_{\mu}, \lambda] = 0, \quad (3.89)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.90)$$

Written explicitly in form of a matrix equation the (3.89) reads

$$\partial_\rho \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.91)$$

$$\partial_+ \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \xi & \frac{2\pi(4\mathcal{L}_1-\mathcal{L}_2)}{3c} & 0 & 0 & -\frac{2i\pi(\mathcal{L}_1-\mathcal{L}_2)}{3c} & 0 \\ 1 & \xi & 0 & 0 & 0 & -\frac{2i\pi(\mathcal{L}_1-\mathcal{L}_2)}{c} \\ -\frac{4i\pi(\mathcal{L}_1-\mathcal{L}_2)}{3c} & 0 & \xi & \frac{2\pi(2\mathcal{L}_1+\mathcal{L}_2)}{3c} & 0 & 0 \\ 0 & -\frac{4i\pi(\mathcal{L}_1-\mathcal{L}_2)}{3c} & 3 & \xi & \frac{4\pi(2\mathcal{L}_1+\mathcal{L}_2)}{3c} & 0 \\ 0 & 0 & 0 & 2 & \xi & \frac{2\pi(2\mathcal{L}_1+\mathcal{L}_2)}{c} \\ 0 & 0 & 0 & 0 & 1 & \xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.92)$$

$$\partial_- \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} -\xi & 0 & 0 & 0 & 0 & 0 \\ 0 & -\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & -\xi & 0 \\ 0 & 0 & 0 & 0 & 0 & -\xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.93)$$

The solutions to these equations are given as

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \mathcal{R}(\rho) f_+(x_+) f_-(x_-), \quad (3.94)$$

where, $\mathcal{R}(\rho)$ is the square matrix in (3.6) which contains the ρ -dependence. The x_+ and x_- dependent pieces are as follows

$$\begin{aligned} f_+(x_+) = & c_1 e^{-(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + \xi)x_+} y_1 + c_2 e^{-(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + \xi)x_+} y_2 + c_3 e^{-(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi)x_+} y_3 \\ & + c_4 e^{-(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} - 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi)x_+} y_4 + c_5 e^{-(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} - 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi)x_+} y_5 \\ & + c_6 e^{-(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi)x_+} y_6, \end{aligned} \quad (3.95)$$

y_i are the eigenvectors of the matrix that appears in the $+$ component of the Killing spinor equation.

$$f_-(x_-) = e^{\xi x_-}. \quad (3.96)$$

The value of the $u(1)$ field for which we get the proper periodicity of the spinor is

$$\xi = \pm \left(\sqrt{\frac{2\pi\mathcal{L}_1}{k}} \pm \frac{1}{2} \sqrt{\frac{2\pi\mathcal{L}_2}{k}} \right) \quad \text{or,} \quad \xi = \pm \frac{1}{2} \sqrt{\frac{2\pi\mathcal{L}_2}{k}}. \quad (3.97)$$

For the case of the black holes in this paper we have explicitly solved the Killing spinor components of G_r^+ and U_r^+ . The same method can be employed to solve for the components of the G_r^- and U_r^- generators as well. Note that for the higher spin black holes we solved the Killing spinor equations with $\mathcal{W} = \mu = 0$. One can in principle solve the Killing spinor equation with these turned on.

4. Supersymmetry and holonomy

In this section we show that the periodicity conditions for the Killing spinor which we have obtained by carefully solving the equations explicitly can be written in terms of a condition on the holonomy of the gauge connection around the angular ϕ direction. We state the condition for a general gauge connection belonging to the $sl(N|N-1)$ superalgebra. We show that whenever the holonomy of the $u(1)$ part of the connection along with eigenvalues of the rest of the background holonomy weighted with the odd roots of the superalgebra becomes trivial then the Killing spinor is periodic. We then explicitly verify that this condition reproduces the equations (3.30) and (3.31) we find for the higher spin conical defects in the $sl(3|2)$ algebra. We then proceed to combine the supersymmetry requirement along with the requirement that the holonomy is smooth to check if higher spin supersymmetric conical defects are smooth in the $sl(3|2)$ theory.

4.1 Killing spinor periodicity as a holonomy

The equation for the covariantly conserved Killing spinor satisfies the equation given by

$$\mathcal{D}_\mu \epsilon \equiv \partial_\mu \epsilon + [A_\mu, \epsilon] = 0. \quad (4.1)$$

Here $\epsilon = \epsilon^i G_i$ is a linear combination of the fermionic generators. $A_\mu = A_\mu^a T_a$ are the connection one forms valued in the bosonic part of the algebra. It is convenient to choose the fermionic generators in the Cartan-Weyl basis of the superalgebra. For definiteness we can work with the supergroup $sl(N|N-1)$ but the discussion can be easily generalized to any superalgebra. In the Cartan-Weyl basis, the generators satisfy the following conditions: let H_r be the Cartan's of the superalgebra and J be the $U(1)$. Then we have the commutation relations

$$[H_r, G_i] = \alpha_i^r G_i, \quad [J, G_i] = \pm G_i, \quad (4.2)$$

where α_i^r is the r th component of the odd root α_i . As mentioned in section 2 we see that the integrability condition for the Killing spinor equation is satisfied since the background gauge field satisfies the equation of motion. We can therefore solve the equation in (4.1) formally by writing the solution as

$$\epsilon(x) = \mathcal{P}(e^{\int_{x_0}^x A_\mu dx^\mu}) \hat{\epsilon}(x_0) \mathcal{P}(e^{-\int_{x_0}^x A_\mu dx^\mu}), \quad (4.3)$$

where x_0 is a base point and $\hat{\epsilon}(x_0)$ is a constant spinor and \mathcal{P} refers to the path ordered exponential. To determine whether the spinor is periodic we can consider $x = (\rho, t, 2\pi)$ and $x_0 = (\rho, t, 0)$ and the integral is along the constant time circle in the angular direction. For all the solutions considered in this paper, the holonomy along this circle reduces to the form

$$\text{Hol}_\phi(A) = \exp\left(\oint A_\mu dx^\mu\right) = b^{-1}(\rho) \exp\left(\oint a_\phi d\phi\right) b(\rho), \quad (4.4)$$

where $b(\rho)$ is the matrix which contains the ρ dependence. The connection a_ϕ is constant and can be easily integrated. Since it is a sum of the bosonic generators we can write it as

$$\exp\left(\oint a_\phi d\phi\right) = S^{-1} \exp(2\pi(\lambda^r H_r + 2\xi J)) S, \quad (4.5)$$

where S is the similarity transformation which brings the constant holonomy in the diagonal form. Now substituting the equation (4.5) in the solution of the Killing spinor given in (4.3) we find the periodicity of the spinor is determined by

$$\epsilon(\rho, t, 2\pi) = b^{-1} S^{-1} e^{2\pi(\lambda^r H_r + \xi J)} S b(\rho) \hat{\epsilon}(\rho, t, 0) b^{-1}(\rho) S^{-1} e^{-2\pi(\lambda^r H_r + 2\xi J)} S b. \quad (4.6)$$

Since the Cartan-Weyl basis G^i for fermionic generators is complete we have the relation

$$S b(\rho) \hat{\epsilon}(\rho, t, 0) b^{-1}(\rho) S^{-1} = \epsilon(\rho, t, 0) = \tilde{\epsilon}^i(\rho, t, 0) G_i. \quad (4.7)$$

From the commutation relations given in (4.2) we find

$$e^{2\pi(\lambda^r H_r + \zeta J)} G^i e^{-2\pi(\lambda^r H_r + 2\xi J)} = e^{2\pi(\lambda^r \alpha_i^r \pm 2\xi)} G_i. \quad (4.8)$$

Now substitute equations (4.7) and (4.8) into the periodicity equation for the Killing spinor given in (4.6) and consider the case in which say any one of the $\tilde{\epsilon}^i$ is turned on and the rest set to zero. Then we see that the spinor with $\tilde{\epsilon}^i$ along the generator G^i is periodic if the following condition is true

$$\lambda^r \alpha_i^r \pm 2\xi = in. \quad (4.9)$$

where n is any integer and r is summed over the Cartan directions other than the $U(1)$. Note that the sign \pm depends on the sign of the commutation relation $[J, G^i] = \pm G^i$. Thus we find that periodicity property of the Killing spinor along the ϕ direction can be generally stated in terms of product of the holonomies of the background connection with the odd roots of the superalgebra. The number of supersymmetries preserved can also be found easily by checking how many among all the fermionic directions labelled by i satisfy the condition (4.9).

We will now verify the general equation we have derived in (4.9) for the specific situation of higher spin conical defects in the $sl(3|2)$ theory. From the gauge connection in (3.12) we obtain

$$a_\phi = \delta_{-1} T_{-1}^+ + \delta_1 T_1^+ + \beta_{-1} T_{-1}^- + \beta_1 T_1^- + \eta_{-1} W_{-1} + \eta_1 W_1 + 2\xi J. \quad (4.10)$$

We now use representation of the matrices for $sl(3)$ given in [18] with $\sigma = (\frac{3}{4})^2$ and the following representation for $sl(2)$ in terms of the Pauli matrices

$$T_1^+ = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad T_{-1}^+ = \frac{1}{2}(\sigma_1 + i\sigma_2), \quad T_0^+ = \frac{1}{2}\sigma_3. \quad (4.11)$$

Then the eigenvalues of the $sl(3) \oplus sl(2)$ part of the matrix a_ϕ along with the $u(1)$ part is given by

$$Sa_\phi S^{-1} = \text{Diag} \left[2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)}, 0, -2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)}, i\sqrt{\alpha}\delta, -i\sqrt{\alpha}\delta \right] + 2\xi J. \quad (4.12)$$

We will now write this as a linear combination of the Cartan's of $sl(3|2)$. From the appendix which lists the generators of $sl(N|N-1)$ we find that (4.12) can be written as

$$Sa_\phi S^{-1} = 2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)}(H_1 + H_2) + i\sqrt{\alpha}\delta H_4 + 2\xi J, \quad (4.13)$$

where the Cartan matrices are given by

$$H_1 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \\ & & & & 0 \end{pmatrix} \quad H_2 = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & -1 & \\ & & & 0 \\ & & & & 0 \end{pmatrix} \quad H_4 = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \\ & & & & -1 \end{pmatrix}. \quad (4.14)$$

In this representation the $U(1)$ generator J is given by

$$J = \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \quad (4.15)$$

We now need the odd roots of the supercharges with J charge 1. In the Cartan-Weyl basis these are given by 6 matrices $E_{\bar{i},k}$ with $\bar{i} = \bar{4}, \bar{5}$ and $k = 1, 2, 3$. They correspond to the 6 generators $G_{\pm 1/2}^+, U_{\pm 1/2}^+, U_{\pm 3/2}^+$. Evaluating the commutation relations explicitly using the matrix representation given in the appendix we find the following roots

$$\begin{aligned} [H_1 + H_2, E_{\bar{i}1}] &= -E_{\bar{i}1}, & [H_1 + H_2, E_{\bar{i}2}] &= 0, & [H_1 + H_2, E_{\bar{i}3}] &= E_{\bar{i}3}, \\ [H_{\bar{4}}, E_{\bar{4}k}] &= E_{\bar{4}k}, & [H_{\bar{4}}, E_{\bar{5}k}] &= -E_{\bar{5}k}. \end{aligned} \quad (4.16)$$

We now have all the information to derive the supersymmetric conditions given in (4.9). Consider the supercharge $E_{\bar{4}1}$, using the holonomy of the background given in (4.12) and the roots from (4.16) we find the following condition

$$-2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)} + i\sqrt{\alpha}\delta + 2\xi = in. \quad (4.17)$$

We see that this matches with one of equations in (3.31). Now consider the supercharge $E_{\bar{4}2}$, again using (4.12) and (4.16) we obtain

$$i\sqrt{\alpha}\delta + 2\xi = in. \quad (4.18)$$

This coincides with one of the equations in (3.30). Repeating this explicitly for all the remaining supercharges we obtain the 6 conditions in (3.30) and (3.31).

4.2 Smooth holonomy and supersymmetry

To find the allowed supersymmetric conical defects we will impose the smoothness condition on the holonomy and also require the background holonomy to be supersymmetric. Demanding the holonomy around the angular direction is trivial leads to the following condition

$$\text{Hol}_\phi(A) = \mathbf{1}_{3 \times 3} \otimes \pm \mathbf{1}_{2 \times 2} \otimes \mathbf{1}_{1 \times 1}. \quad (4.19)$$

We then have following conditions on δ , β and η .

$$\begin{aligned} \sqrt{\alpha}\delta &= \pm \frac{m}{2}, \\ 2\sqrt{\alpha}(\beta^2 - (\frac{3}{4}\eta)^2)^{1/2} &= \pm p, \\ -2i\xi &= \pm q. \end{aligned} \quad (4.20)$$

where $n, p, q \in \mathbb{Z}$. Substituting the periodicity conditions of the Killing spinors given in (3.30) and (3.31) we obtain the following conditions

$$q \mp \frac{m}{2} = n, \quad q \mp \left(\frac{m}{2} \pm p\right) = n. \quad (4.21)$$

Let us now examine if any of the conical defects in the $sl(3|2)$ theory satisfy the requirement that they lie in the domain as pointed out in [19]

$$-\frac{c}{24} < L_0 < 0, \quad (4.22)$$

where c is the central charge of the theory which can be written in terms of the cosmological constant. For the $sl(3|2)$ theory L_0 in terms of the holonomy is given by

$$L_0 = \frac{c}{24\epsilon_{(3|2)}} (\text{str}(a_\phi^2)) \quad (4.23)$$

Note that by defining L_0 as given in (4.23) the shift in energy due to the presence of the presence of the $U(1)$ field [35, 26] is accounted for. Substituting the values of the holonomy from (4.20) in (4.23) we obtain the following bound that the integers p, q, m must satisfy

$$0 < p^2 - \left(\frac{m}{2}\right)^2 - 6q^2 < \frac{3}{4}. \quad (4.24)$$

The factor of 6 occurs on taking the super trace of J^2 using the definition of J given in (4.15). There are no conical defects which satisfy the above bound. However to illustrate the method of figuring out the allowed supersymmetric defects for the general $sl(N|N-1)$ we proceed to study the susy conditions. On substituting for $m/2$ from the supersymmetric holonomy conditions given in (4.21) we obtain the following bounds

$$0 < p^2 - (q - n)^2 - 6q^2 < 3/4, \quad 0 < p^2 - (q - n \mp p)^2 - 6q^2 < 3/4. \quad (4.25)$$

It is clear that any of the above bounds are satisfied since there is no integer between 0 and 3/4. Thus there are no supersymmetric smooth conical defects in the $sl(3|2)$.

This analysis can be easily extended for the $sl(N|N-1)$ theory and it will be interesting to find if there exists supersymmetric smooth conical defects. These solutions should correspond to chiral primaries in the dual theory.

5. Conclusions

We have constructed a class of conical defects and black holes in the $sl(3|2)$ theory. These solutions in general have fields in $sl(3) \oplus sl(2) \oplus u(1)$ directions turned on. We have developed a general method to solve the Killing spinor equations for these solutions and obtained the conditions under which they are supersymmetric explicitly. Observing the conditions for the Killing spinor to be periodic in the angular

directions we show that for the general $sl(N|N-1)$ theory these conditions can be formulated invariantly in terms of the products of the background holonomies with the odd roots of the superalgebra. This condition is given in (4.9). We have compared this condition with the conditions obtained by explicitly solving the Killing spinor equations for the conical defects constructed in this paper and shown that they are in agreement. The condition for the Killing spinor to be supersymmetric can be easily generalized for solutions in Chern-Simons theories based on any super group.

It will be interesting to generalize the conical defects constructed in this paper explicitly for the $sl(3|2)$ theory to the $sl(N|N-1)$ theory and study their supersymmetry along with the smoothness conditions as we have done for the case of $sl(3|2)$ theory. We have shown in this paper that for the $sl(3|2)$ theory there are no smooth conical defects. However the $sl(N|N-1)$ theory might admit smooth conical defects which are also supersymmetric [36]. These solutions are possible candidates for chiral primaries in the dual CFT. It will be interesting to classify them and compare them with the chiral primaries of the dual Kazama-Suzuki model conjectured to be the large N limit of these theories [14]. Conical surplus solutions in the bosonic $sl(N, C)$ Chern-Simons theory have been shown to agree with the light states of the dual minimal model [19]. It will be interesting to see if the Euclidean supersymmetric version of the Chern-Simons theory studied in this paper admits conical surplus solutions and check if they are supersymmetric. One can then verify if they correspond to possible light states in the dual Kazama-Suzuki model of [14]¹.

The black holes we constructed in the $sl(3|2)$ theory have in addition to fields in $sl(3)$ also fields in the extra $sl(2)$ turned on. It will be interesting to study the thermodynamic properties of these black hole solutions and obtain an expression for their partition function both from the bulk theory and the CFT.

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A. Cartan-Weyl basis for $sl(N|N-1)$

One can construct following [33] a basis of matrices for the $sl(N|N-1)$ algebra. Let's consider $(2N-1)^2$ matrices e_{IJ} of order $2N-1$ so that $(e_{IJ})_{KL} = \delta_{IL}\delta_{JK}$

¹We thank Rajesh Gopakumar for discussions regarding this point

$(I, J, K, L = 1, \dots, 2N - 1)$ and define the $(2N - 1)^2 - 1$ generators

$$E_{ij} = e_{ij} - \delta_{ij}(e_{kk} + e_{\bar{k}\bar{k}}), \quad E_{i\bar{j}} = e_{i\bar{j}}, \quad (\text{A.1})$$

$$E_{\bar{i}j} = e_{\bar{i}j} + \delta_{\bar{i}j}(e_{kk} + e_{\bar{k}\bar{k}}), \quad E_{\bar{i}\bar{j}} = e_{\bar{i}\bar{j}}, \quad (\text{A.2})$$

where i, j, \dots run from 1 to N and \bar{i}, \bar{j}, \dots from $N + 1$ to $2N - 1$.

The generators for the various subalgebras of $sl(3|2)$ are as follows

- $u(1) : Z = E_{kk} = -E_{\bar{k}\bar{k}} = -((N - 1)e_{kk} + Ne_{\bar{k}\bar{k}}).$
- $sl(N) : E_{ij} - \frac{1}{N}\delta_{ij}Z.$
- $sl(N - 1) : E_{\bar{i}\bar{j}} + \frac{1}{N-1}\delta_{\bar{i}\bar{j}}Z.$
- $(\bar{N}, N - 1)$ representation of $sl(N) \oplus sl(N - 1) \oplus u(1) : E_{i\bar{j}}.$
- $(N, \overline{N - 1})$ representation of $sl(N) \oplus sl(N - 1) \oplus u(1) : E_{\bar{i}j}.$

In the Cartan-Weyl basis, the generators are given by

- Cartan subalgebra

$$H_i = E_{ii} - E_{i+1,i+1}, \quad \text{for } 1 \leq i \leq N - 1, \quad (\text{A.3})$$

$$H_{\bar{i}} = E_{\bar{i}\bar{i}} - E_{\bar{i}+1,\bar{i}+1}, \quad \text{for } N + 1 \leq i \leq 2N - 2, \quad (\text{A.4})$$

$$H_N = E_{NN} + E_{N+1,N+1}. \quad (\text{A.5})$$

- Raising operators

$$E_{ij} \text{ with } i < j \text{ for } sl(N), \quad E_{i\bar{j}} \text{ with } \bar{i} < \bar{j} \text{ for } sl(N - 1), \quad E_{i\bar{j}}. \text{ for the odd part} \quad (\text{A.6})$$

- Lowering operators

$$E_{ji} \text{ with } i < j \text{ for } sl(N), \quad E_{\bar{j}\bar{i}} \text{ with } \bar{i} < \bar{j} \text{ for } sl(N - 1), \quad E_{\bar{i}j}. \text{ for the odd part} \quad (\text{A.7})$$

The commutation relations in this basis are

$$\begin{aligned} [H_I, H_J] &= 0, \\ [H_K, E_{IJ}] &= \delta_{IK}E_{KJ} - \delta_{I,K+1}E_{K+1,J} - \delta_{KJ}E_{IK} + \delta_{K+1,J}E_{I,K+1} \quad (K \neq N), \\ [H_N, E_{IJ}] &= \delta_{Im}E_{NJ} + \delta_{I,N+1}E_{N+1,J} - \delta_{NJ}E_{Im} - \delta_{N+1,J}E_{I,N+1}, \\ [E_{IJ}, E_{KL}] &= \delta_{JK}E_{IL} - \delta_{IL}E_{KJ} \quad \text{for } E_{IJ} \text{ and } E_{KL} \text{ even}, \\ [E_{IJ}, E_{KL}] &= \delta_{JK}E_{IL} - \delta_{IL}E_{KJ} \quad \text{for } E_{IJ} \text{ even and } E_{KL} \text{ odd}, \\ \{E_{IJ}, E_{KL}\} &= \delta_{JK}E_{IL} + \delta_{IL}E_{KJ} \quad \text{for } E_{IJ} \text{ and } E_{KL} \text{ odd}. \end{aligned} \quad (\text{A.8})$$

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